Physics Letters B 666 (2008) 517–521

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Fermions tunnelling from GHS and non-extremal D1–D5 black holes

Qing-Quan Jiang

Institute of Particle Physics, Central China Normal University, Wuhan, Hubei 430079, China

ARTICLE INFO

Article history:
Received 19 June 2008
Received in revised form 31 July 2008
Accepted 4 August 2008
Available online 7 August 2008
Editor: M. Cvetič

Keywords:
04.70.Dy
04.62.+v

Keywords:
Fermions
Hawking radiation
Tunnelling
String black hole

ABSTRACT

Recent research shows that fermions tunnelling can result in correct Hawking temperature of a black hole. In this letter, choosing a set of appropriate matrices γ^μ, we attempt to study Hawking radiation of Dirac particles across the horizons of the GHS and non-extremal five-dimensional D1–D5 black holes in string theory by using fermions tunnelling method. Finally, the expected Hawking temperatures of the GHS and non-extremal D1–D5 black holes are correctly recovered.

1. Introduction

In 1974, Hawking proved that a black hole can radiate particles with thermal spectrum at the temperature $T = \kappa/2\pi$ [1], where κ is the surface gravity of the horizon. Since then, although many methods have appeared to correctly derive Hawking radiation of a black hole, not all of them are satisfying in the process of extending these methods. In 2000, Parikh and Wilczek, elaborating Kraus and Wilczek's work [2,3], presented another new derivation of Hawking radiation, where Hawking radiation is treated as a quantum tunnelling process [4]. A pair of particles is spontaneously created just inside the horizon as a result of quantum vacuum fluctuations near the horizon, classically the positive energy particle do not escape out along the classically forbidden region, but quantum mechanically it can tunnel out to the infinity. For the tunnelling picture, the Hawking temperature is directly related to the imaginary part of the action of particles tunnelling from inside to outside horizon along the classically forbidden region. In [4], derivation of the imaginary part of the action mainly depends on the integration of the radial momentum p_r for the emitted particles, which is normally called as the Null Geodesic method. The other method, appeared in [5], regards the action of the emitted particles across the classically forbidden region satisfies the relativistic Hamilton–Jacobi equation, and solving it yields the imaginary part of the action, which is an extension of the complex path analysis proposed by Padmanabhan et al. [6,7]. Till now, the tunnelling method has been already proved very robust for scalar particles across black hole horizons, and successfully recovered the correct Hawking temperatures of a wide variety of interesting and exotic spacetimes [8–19].

Recently, Kerner and Mann, modelling Hawking radiation as fermions tunnelling, have successfully recovered the Unruh and Hawking temperatures for the Rindler spacetime and general nonrotating black hole [20]. In the model, choosing a set of appropriate matrices γ^μ is an important technique, or we may not correctly recover the Hawking temperature we expected. And near the horizon, the imaginary part of the action is determined by the covariant Dirac equation. To further show the robustness of fermions tunnelling method, many recent papers appear to discuss Hawking radiation of Dirac particles via tunnelling from $(2+1)$-dimensional BTZ black hole [21], dynamical horizons [22], Kerr and Kerr–Newman black holes [23,24], charged dilatonic black holes [25] and rotating black holes in de Sitter spaces [26]. However these involved black holes share in taking 3- or 4-dimensional spacetimes. In this letter, by considering the Garfinkle–Horowitz–Strominger (GHS) [27] and charged non-extremal 5-dimensional D1–D5 black holes [28] in string theory, we once again confirm fermions tunnelling method. It is expected that our result strengthens the validity and power of the method.

The letter is outlined as follows. In Section 2, we begin with our studies by applying fermions tunnelling method to study Hawking radiation of Dirac particles across the GHS black hole. Section 3 is focus on fermions tunnelling from the charged non-extremal

E-mail address: jiangqq@iopp.ccnu.edu.cn.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2008.08.005

5-dimensional D1–D5 black hole. Section 4 ends up with some conclusions and discussions.

2. Fermions tunnelling from GHS black hole

In this section, we are devoted to study Hawking radiation of Dirac particles across the horizon of the GHS black hole in string theory by using fermions tunnelling method. The GHS black hole is a member of a family of solutions to low energy string theory described by the action

$$S = \int d^4x \sqrt{-g} e^{-2\phi}\left[-R - 4(\nabla\phi)^2 + F^2\right], \tag{1}$$

where ϕ is the dilaton field and $F_{\mu\nu}$ is the maxwell field associated with a $U(1)$ subgroup of $E_8 \times E_8$ or $\mathrm{Spin}(32)/Z_2$. The GHS black hole in the string frame is then given by

$$ds^2_{\text{string}} = -f(r)\,dt^2 + \frac{1}{h(r)}dr^2 + r^2\,d\Omega, \tag{2}$$

where

$$f(r) = \left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right)^{-1},$$

$$h(r) = \left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right).$$

Here ϕ_0 is the asymptotic constant value of the dilaton field. For $Q^2 < 2e^{-2\phi_0}M^2$, the metric (2) describes a black hole with an event horizon located at $r_h = 2Me^{\phi_0}$.

Now we focus on studying fermions tunnelling from the GHS black hole. The motion equation of Dirac fields Ψ in the curved spacetimes satisfies the following covariant Dirac equation as

$$i\gamma^\mu D_\mu \Psi + \frac{m}{\hbar}\Psi = 0, \tag{3}$$

where $D_\mu = \partial_\mu + \frac{i}{2}\Gamma^{\alpha\beta}_\mu \Sigma_{\alpha\beta}$ is the spin covariant derivative, $\Sigma_{\alpha\beta} = \frac{i}{4\pi}[\gamma^\alpha, \gamma^\beta]$ and m is the mass of the emitted particles. The matrices γ^μ is determined by $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times I$, where I is the identity matrix. In our case, we choose the matrices γ^μ for the GHS spacetime taking the form as

$$\gamma^t = \frac{1}{\sqrt{f(r)}}\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \gamma^r = \sqrt{h(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix},$$
$$\gamma^\theta = \frac{1}{r}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \qquad \gamma^\varphi = \frac{1}{r\sin\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \tag{4}$$

and the σ^i $(i = 1, 2, 3)$ are the Pauli matrices, which are respectively described by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5}$$

Dirac fields with the spin 1/2 have two spin states, that is the spin up and spin down states. When measuring the spin states along the r direction, the spin up state takes the same direction as r, but the spin down case has the opposite direction. In this letter, we only consider Dirac particles with the spin up without loss of generality, because after a manner fully analogous to the spin up case the same result will be present for Dirac particles with the spin down. Now we employ the following ansatz for Dirac particles with the spin up

$$\Psi_\uparrow(t, r, \theta, \varphi) = \begin{pmatrix} A(t, r, \theta, \varphi)\xi_\uparrow \\ B(t, r, \theta, \varphi)\xi_\uparrow \end{pmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t, r, \theta, \varphi)\right]$$
$$= \begin{pmatrix} A(t, r, \theta, \varphi) \\ 0 \\ B(t, r, \theta, \varphi) \\ 0 \end{pmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t, r, \theta, \varphi)\right], \tag{6}$$

where $\xi_\uparrow = \binom{1}{0}$ is the eigenvector of the matrix σ^3, and the corresponding eigenvalue is 1, which describes Dirac particles with the spin up. Applying the WKB approximation, and inserting the ansatz (6) into the covariant Dirac equation (3), and then dividing by the exponential term and multiplying by $\hbar$, the resulting equations to leading order in $\hbar$ take the forms as

$$B\left(\frac{1}{\sqrt{f(r)}}\partial_t I_\uparrow + \sqrt{h(r)}\,\partial_r I_\uparrow\right) - mA = 0, \tag{7}$$

$$B\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\varphi I_\uparrow\right) = 0, \tag{8}$$

$$A\left(\frac{1}{\sqrt{f(r)}}\partial_t I_\uparrow - \sqrt{h(r)}\,\partial_r I_\uparrow\right) + mB = 0, \tag{9}$$

$$A\left(\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\varphi I_\uparrow\right) = 0. \tag{10}$$

Here the contributions of the derivatives A and B and the components $\Gamma^{\alpha\beta}_\mu \Sigma_{\alpha\beta}$ have already been neglected to the lowest order in WKB approximation due to the fact that they are all of order $\mathcal{O}(\hbar)$. To carry out the separation of variables for the above equations, considering the symmetries of the GHS spacetime we employ the ansatz

$$I_\uparrow = -\mathcal{E}t + \mathcal{J}(\theta, \phi) + \mathcal{W}(r), \tag{11}$$

where $\mathcal{E}$ is the energy of the emitted Dirac particle. Near the horizon, substituting the ansatz (11) into Eqs. (7), (8), (9) and (10) yields

$$B\left(-\frac{\mathcal{E}}{\sqrt{f_{,r}(r_h)(r - r_h)}} + \sqrt{h_{,r}(r_h)(r - r_h)}\,\partial_r\mathcal{W}(r)\right) - mA = 0, \tag{12}$$

$$B\left(\frac{1}{r}\partial_\theta\mathcal{J}(\theta, \phi) + \frac{i}{r\sin\theta}\partial_\phi\mathcal{J}(\theta, \phi)\right) = 0, \tag{13}$$

$$A\left(-\frac{\mathcal{E}}{\sqrt{f_{,r}(r_h)(r - r_h)}} - \sqrt{h_{,r}(r_h)(r - r_h)}\,\partial_r\mathcal{W}(r)\right) + mB = 0, \tag{14}$$

$$A\left(\frac{1}{r}\partial_\theta\mathcal{J}(\theta, \phi) + \frac{i}{r\sin\theta}\partial_\phi\mathcal{J}(\theta, \phi)\right) = 0, \tag{15}$$

where $f_{,r}$ and $h_{,r}$ denote a derivative with respect to r. Careful analysis on the above equations, we find $\mathcal{J}$ must be a complex function, which means it will yield a contribution to the imaginary part of the action. However further studying shows that the contribution of $\mathcal{J}$ is completely the same for both the outgoing and ingoing solutions, and therefore its total contribution to the tunnelling rate is cancelled out when dividing the outgoing probability by the ingoing probability. Then it is no need to solve the equations about the complex function $\mathcal{J}$. Now our attention should be focus on the radial function $\mathcal{W}$. From Eqs. (12) and (14), there will be a non-trivial solution for A and B if and only if the determinant of the coefficient matrix vanishes, which results

$$\partial_r\mathcal{W}(r) = \pm\frac{\sqrt{\mathcal{E}^2 + f_{,r}(r_h)(r - r_h)m^2}}{\sqrt{f_{,r}(r_h)h_{,r}(r_h)}(r - r_h)}. \tag{16}$$

Integrating the pole at the horizon of the GHS black hole as in Refs. [16,17], we have

$$\mathcal{W}_\pm = \pm i\pi\frac{\mathcal{E}}{\sqrt{f_{,r}(r_h)h_{,r}(r_h)}}. \tag{17}$$

In Eq. (17), the $+/-$ sign corresponds to the outgoing/incoming solutions. The WKB approximation tells us the tunnelling probability is related to the imaginary part of the action as $P = \exp[-\frac{2}{\hbar}\operatorname{Im} I]$, where I is the action of particles across the black hole horizon. Set $\hbar$ to unity, and the overall tunnelling rate can be written as

$$\begin{aligned}\Gamma &= \frac{P_{\rm out}}{P_{\rm in}} = \frac{\exp[-2(\operatorname{Im}\mathcal{W}_+ + \operatorname{Im}\mathcal{J})]}{\exp[-2(\operatorname{Im}\mathcal{W}_- + \operatorname{Im}\mathcal{J})]}\\ &= \exp\left[-\frac{4\pi\mathcal{E}}{\sqrt{f_{,r}(r_h)h_{,r}(r_h)}}\right],\end{aligned} \tag{18}$$

which results in Hawking temperature of the GHS black hole taking

$$T = \frac{\sqrt{f_{,r}(r_h)h_{,r}(r_h)}}{4\pi} = \frac{1}{8\pi M e^{\phi_0}}. \tag{19}$$

The derivation of Hawking temperature of the GHS black hole via fermions tunneling method takes the same form as that by using anomalous cancelation method [29]. However, in Ref. [30], the authors have derived an Hawking temperature different from Eq. (19), but having the important property to vanish in the extremal limit. This mainly depends on where the observers live. In our case, the Hawking temperature is measured by static observers at infinity. In Ref. [30] the Hawking temperature is measured by preferred observers just outside the horizon. In the above discussion, we only consider Dirac fields with the spin up. For the spin-down case, we can proceed in a manner fully analogous to the spin up case to recover the same result. On the other hand, the tunneling rate (18) is derived by having neglected the higher terms about $\mathcal{E}$, and therefore the resulting thermal spectrum is purely thermal. In fact, the factual tunneling picture is that when the positive energy particle created by just inside the horizon as a result of quantum vacuum fluctuations near the horizon tunnels out to the infinity to form Hawking radiation, the negative energy partner remains behind and effectively lowers the mass of the black hole. That means when studying Hawking radiation of a black hole, we should consider the fluctuation of the spacetime geometry—namely, the energy conservation. In such case, the emitted spectrum is no longer purely thermal, and contains the higher terms about $\mathcal{E}$. In this letter, we still consider the fixed spacetime background. In the following section, to further check the validity of fermions tunnelling method, we take the non-extremal five-dimensional D1–D5 black hole in string theory as an example to study Hawking radiation of Dirac particles via tunnelling from the horizon.

3. Fermions tunnelling from non-extremal D1–D5 black hole

As another example of fermions tunnelling method, we consider a non-extremal five-dimensional black hole, which is originated from a brane configuration in Type IIB superstring compactified on $S^1 \times T^4$. In our case, the configuration is composed of D1-branes wrapping S^1, D5-branes wrapping $S^1 \times T^4$, and momentum modes along S^1. In this configuration, the solution of Type IIB supergravity is a supersymmetric background known as the extremal five-dimensional D1–D5 black hole. The extremal black hole has zero Hawking temperature, and therefore no Hawking radiation will be present at the infinity. Hence to consider Hawking radiation we should study the non-extremal D1–D5 black hole.

The ten-dimensional supergravity background corresponding to the non-extremal D1–D5 black hole has the following form in the string frame [28]

$$\begin{aligned} ds_{10}^2 &= f_1^{-1/2} f_5^{-1/2}\left(-h f_n^{-1}\,dt^2 + f_n\left(dx_5 + \left(1-\tilde f_n^{-1}\right)dt\right)^2\right)\\ &\quad + f_1^{1/2} f_5^{-1/2}\left(dx_6^2+\cdots+dx_9^2\right) + f_1^{1/2} f_5^{1/2}\left(h^{-1}\,dr^2 + r^2\,d\Omega_3^2\right),\\ e^{-2\phi} &= f_1^{-1} f_5, \qquad C_{05} = \tilde f_1^{-1} - 1,\\ F_{ijk} &= \frac{1}{2}\epsilon_{ijkl}\partial_l \tilde f_5, \quad i,j,k,l = 1,2,3,4, \end{aligned} \tag{20}$$

where x_5 and $x_6,\dots,x_9$ are periodic coordinate along S^1 and T^4, and F is the three-form field strength of the RR two-form gauge potential C, $F = dC$. In Eq. (20), various functions can be given by coordinates $x_1,\dots,x_4$ as

$$\begin{aligned} h &= 1 - \frac{r_0^2}{r^2}, \qquad f_{1,5,n} = 1 + \frac{r_{1,5,n}^2}{r^2},\\ \tilde f_{1,n}^{-1} &= 1 - \frac{r_0^2 \sinh\alpha_{1,n}\cosh\alpha_{1,n}}{r^2} f_{1,n}^{-1},\\ r_{1,5,n}^2 &= r_0^2\sinh^2\alpha_{1,5,n}, \qquad r^2 = x_1^2+\cdots+x_4^2, \end{aligned}$$

where h and $f_{1,5,n}$ are harmonic functions, respectively, representing the non-extremality, and the presence of D1 and D5, and momentum modes. Following the procedure of [32,33] to carry on a dimensional reduction of Eq. (20) along $S^1 \times T^4$, we can obtain the Einstein metric of the non-extremal five-dimensional black hole

$$ds_5^2 = -\lambda^{-2/3} h\,dt^2 + \lambda^{1/3}\left(h^{-1}\,dr^2 + r^2\,d\Omega_3^2\right), \tag{21}$$

where $d\Omega_3^2$ is the metric on the unit 3-sphere, and λ is defined by $\lambda = f_1 f_5 f_n$. The event horizon of the black hole is located at $r = r_h = r_0$. Upon a dimensional reduction, three kinds of gauge fields will be present. One is the Kaluza–Klein gauge field $\mathcal{A}_\mu^{(K)}$, which is originating from the metric. The second gauge field $\mathcal{A}_\mu^{(1)}$ stems from $C_{\mu 5}$. The final gauge field is the two-form gauge field coming from $C_{\mu\nu}$ whose field strength is given by the expression of F in Eq. (20). Though the two-form gauge field gives a nonzero contribution to the full black hole background, it will not play any role in the following discussion, and so can be neglected. From (20), the first two gauge fields can be written as

$$\mathcal{A}^K = -\left(\tilde f_n^{-1} - 1\right)dt, \qquad \mathcal{A}^{(1)} = \left(\tilde f_1^{-1} - 1\right)dt. \tag{22}$$

Now, upon a dimensional reduction technique, the spacetime background we concerned is composed of the metric (21) accompanying with the two gauge fields (22). Next, we focus on study Hawking radiation of the non-extremal 5-dimensional D1–D5 black hole by using fermions tunnelling method.

When Dirac fields coupling to gauge fields in a curved spacetime, its motion equation can be written as

$$i\gamma^\mu\left(D_\mu + \frac{ie}{\hbar}\mathcal{A}_\mu\right)\Psi + \frac{m}{\hbar}\Psi = 0. \tag{23}$$

Now to study Hawking radiation of Dirac particles, we should first introduce a set of appropriate matrices γ^μ for the black hole, or we cannot recover the correct Hawking temperature via fermions tunnelling method. In our case, we choose the matrices γ^μ for the non-extremal 5-dimensional D1–D5 black hole taking the form as

$$\begin{aligned} \gamma^t &= \lambda^{1/3} h^{-1/2}\begin{pmatrix} 0 & I\\ -I & 0\end{pmatrix},\\ \gamma^r &= \lambda^{-1/6} h^{1/2}\begin{pmatrix} 0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix},\\ \gamma^\theta &= \lambda^{-1/6} r^{-1}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},\\ \gamma^\varphi &= \lambda^{-1/6}(r\sin\theta)^{-1}\begin{pmatrix} 0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix},\\ \gamma^\psi &= \lambda^{-1/6}(r\cos\theta)^{-1}\begin{pmatrix} -I & 0\\ 0 & I\end{pmatrix}, \end{aligned} \tag{24}$$

where σ^1, σ^2 and σ^3 are the Pauli matrices, and are explicitly given by Eq. (5). As in Section 2, we only consider Dirac particle with the spin up, and employ the ansatz (6) with the variables (t, r, θ, φ) substituted by $(t, r, \theta, \varphi, \psi)$. Inserting the ansatz for Dirac particles with the spin up into the covariant Dirac equation (23), and dividing by the exponential term and multiplying by $\hbar$, the resulting equations to the leading order in $\hbar$ are

$$B\big(\lambda^{1/3}h^{-1/2}(\partial_t I_\uparrow + \mathcal{A}_t) + \lambda^{-1/6}h^{1/2}\partial_r I_\uparrow\big) - A\big(m + \lambda^{-1/6}(r\cos\theta)^{-1}\partial_\psi I_\uparrow\big) = 0, \tag{25}$$

$$B\big(\lambda^{-1/6}r^{-1}\partial_\theta I_\uparrow + i\lambda^{-1/6}(r\sin\theta)^{-1}\partial_\varphi I_\uparrow\big) = 0, \tag{26}$$

$$A\big(\lambda^{1/3}h^{-1/2}(\partial_t I_\uparrow + \mathcal{A}_t) - \lambda^{-1/6}h^{1/2}\partial_r I_\uparrow\big) + B\big(m - \lambda^{-1/6}(r\cos\theta)^{-1}\partial_\psi I_\uparrow\big) = 0, \tag{27}$$

$$A\big(\lambda^{-1/6}r^{-1}\partial_\theta I_\uparrow + i\lambda^{-1/6}(r\sin\theta)^{-1}\partial_\varphi I_\uparrow\big) = 0, \tag{28}$$

where

$$\mathcal{A}_t = -\frac{e_1 r_0^2\sinh\alpha_1\cosh\alpha_1}{r^2+r_1^2} + \frac{e_k r_0^2\sinh\alpha_n\cosh\alpha_n}{r^2+r_n^2}. \tag{29}$$

Here the derivatives about A and B, and the components $\Gamma_\mu^{\alpha\beta}\Sigma_{\alpha\beta}$ are all of order $\mathcal{O}(\hbar)$, so have been neglected to the lowest order in WKB approximation. Although the action of Dirac particle with the spin up satisfies Eqs. (25), (26), (27) and (28), from the above equations it is still difficult to derive the action. So it is necessary to carry on a variable separation to the action. Considering the symmetry of the non-extremal 5-dimensional D1–D5 black hole, we introduce the following ansatz for the action of Dirac particle with the spin up

$$I_\uparrow = -\mathcal{E}t + \mathcal{J}(\theta,\varphi,\psi) + \mathcal{W}(r), \tag{30}$$

where $\mathcal{E}$ is the energy of the emitted particle. Inserting the ansatz (30) into Eqs. (25), (26), (27) and (28), and expanding the resulting equations near the event horizon of the non-extremal D1–D5 black hole yields

$$B\bigg(\frac{\lambda^{1/3}(r_0)}{\sqrt{h_{,r}(r_0)(r-r_0)}}\big(-\mathcal{E}+\mathcal{A}_t(r_0)\big) + \lambda^{-1/6}(r_0)\sqrt{h_{,r}(r_0)(r-r_0)}\,\partial_r\mathcal{W}(r)\bigg) - A\big(m+\lambda^{-1/6}(r_0)(r_0\cos\theta)^{-1}\partial_\psi\mathcal{J}\big) = 0, \tag{31}$$

$$B\big(\lambda^{-1/6}(r_0)r_0^{-1}\partial_\theta\mathcal{J} + i\lambda^{-1/6}(r_0)(r_0\sin\theta)^{-1}\partial_\varphi\mathcal{J}\big) = 0, \tag{32}$$

$$A\bigg(\frac{\lambda^{1/3}(r_0)}{\sqrt{h_{,r}(r_0)(r-r_0)}}\big(-\mathcal{E}+\mathcal{A}_t(r_0)\big) - \lambda^{-1/6}(r_0)\sqrt{h_{,r}(r_0)(r-r_0)}\,\partial_r\mathcal{W}(r)\bigg) + B\big(m-\lambda^{-1/6}(r_0)(r_0\cos\theta)^{-1}\partial_\psi\mathcal{J}\big) = 0, \tag{33}$$

$$A\big(\lambda^{-1/6}(r_0)r_0^{-1}\partial_\theta\mathcal{J} + i\lambda^{-1/6}(r_0)(r_0\sin\theta)^{-1}\partial_\varphi\mathcal{J}\big) = 0. \tag{34}$$

Here $h_{,r}$ represents a derivative with respect to r. From Eqs. (32) and (34), we will find $\mathcal{J}$ is a complex function, at the horizon it yields a contribution to the imaginary part of the action. But further studying indicates that its contributions for the outgoing and ingoing solutions share the same. When the overall tunnelling rate is defined by dividing the outgoing probability by the ingoing probability, its contributions to the overall tunnelling rate will be canceled out. So it is no need to do a careful computation on the complex function $\mathcal{J}$. Now our attention should be focus on the radial function $\mathcal{W}$. And from Eqs. (31) and (33), one can easily see the two equations have a non-trivial solution for A and B if and only if the determinant of the coefficient matrix vanishes, which results in an equation about $\mathcal{W}(r)$. Then integrating the resulting equation at the event horizon yields

$$\mathcal{W}_\pm = \pm i\pi\frac{r_0\cosh\alpha_1\cosh\alpha_5\cosh\alpha_n}{2}(\mathcal{E}+e_1\tanh\alpha_1 - e_K\tanh\alpha_n), \tag{35}$$

where the $+/-$ sign corresponds to the outgoing/incoming solutions, and in Eq. (35) we have considered Eq. (29) at the event horizon. Obviously, the imaginary part of the action is composed of the complex function $\mathcal{J}$ at the horizon and the complex constant $\mathcal{W}_\pm$. According to the WKB approximation, the overall tunnelling rate is given by

$$\Gamma = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\exp[-2(\operatorname{Im}\mathcal{W}_+ + \operatorname{Im}\mathcal{J})]}{\exp[-2(\operatorname{Im}\mathcal{W}_- + \operatorname{Im}\mathcal{J})]} = \exp\big[-2\pi r_0\cosh\alpha_1\cosh\alpha_5\cosh\alpha_n \times(\mathcal{E}+e_1\tanh\alpha_1 - e_K\tanh\alpha_n)\big]. \tag{36}$$

Here we can easily see Hawking temperature of the non-extremal D1–D5 black hole derived by using fermions tunnelling method is

$$T = \frac{1}{2\pi r_0\cosh\alpha_1\cosh\alpha_5\cosh\alpha_n}. \tag{37}$$

This result is exactly equal to that in Refs. [29,31]. In [29,31] Hawking temperature of the non-extremal D1–D5 black hole has been derived by using anomalous cancellation method, but when reducing the higher-dimensional theory to the two-dimensional effective theory they only considered scalar field cases. That means Hawking radiation is determined universally by the horizon properties, and Dirac and scalar particles emit across the horizon at the same Hawking temperature. As in Section 2, in this section we still consider the fixed spacetime background when particles emitted. So fermions tunnelling method successfully recover Hawking temperature of the non-extremal 5-dimensional D1–D5 black hole, its robustness has been proved again.

4. Conclusions and discussions

Hawking radiation of the non-extremal 5-dimensional D1–D5 black hole has been discussed in [29,31] by using anomalous cancellation method. But upon a dimensional reduction technique near the horizon, they only consider scalar field case. So the Hawking temperatures in [29,31] are only for scalar particles emitted across the horizon. Hawking radiation is determined universally by the horizon properties, so Dirac and scalar particles should emit across the horizon at the same Hawking temperature. In this letter, we first introduce a set of appropriate matrices γ^μ for the GHS and non-extremal D1–D5 black hole in the string theory, then study Hawking radiation of Dirac particles across the horizons of the two black holes by using fermions tunnelling method. Finally, the expected Hawking temperatures are correctly recovered.

Fermions tunnelling method have already succeeded in recovering the Hawking temperatures of stationary black holes [20,21,23–26]. For non-stationary black holes, Wu and Cai developed the general tortoise coordinate transformation (GTCT) method to deal with Hawking radiation of Dirac particles, and found a new coupling effect between the spin of the field and the angular momentum of the black hole [34]. In [22], fermions tunnelling from Bardeen–Vaidya and cosmological black holes have been studied, but no coupling effect between the spin of the field and the angular momentum of the black hole is present. That is because the involved non-stationary black holes are of spherical symmetry, and there is no angular momentum. So our next job is to expect fermions tunnelling will recover the new coupling effect for the non-stationary axisymmetric black holes. On the other hand, when particles emitted from the horizon we do not consider the fluctuation of the spacetime geometry. That will result in a purely thermal spectrum. In fact, the factual tunnelling picture is that when the positive energy particle created by just inside the horizon as a result of quantum vacuum fluctuations near the horizon tunnels out to the infinity to form Hawking radiation, the negative energy partner remains behind and effectively lowers the mass of the black hole. That means when studying Hawking radiation of a black hole, we should consider the fluctuation of the spacetime geometry—namely, the energy conservation. In such case, the emitted spectrum is no longer purely thermal, and contains the higher terms about the energy $\mathcal{E}$.

Acknowledgements

This work is supported by National Natural Science Foundation of China with Grant Nos. 10675051, 70571027, 10635020, a grant by the Ministry of Education of China under Grant No. 306022, and a Graduate Innovation Foundation by CCNU.

References

[1] S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.
[2] P. Kraus, F. Wilczek, Nucl. Phys. B 433 (1995) 403, gr-qc/9408003.
[3] P. Kraus, F. Wilczek, Nucl. Phys. B 437 (1995) 231, hep-th/9411219.
[4] M. Parikh, F. Wilczek, Phys. Rev. Lett. 85 (2000) 5042, hep-th/9907001.
[5] M. Angheben, M. Nadalini, L. Vanzo, S. Zerbini, JHEP. 0505 (2005) 014, hep-th/0503081.
[6] K. Srinivasan, T. Padmanabhan, Phys. Rev. D 60 (1999) 024007, gr-qc/9812028.
[7] S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, Class. Quantum Grav. 19 (2002) 2671, qr-qc/0010042.
[8] A.J.M. Medved, Phys. Rev. D 66 (2002) 124009, hep-th/0207247.
[9] S. Hemming, E. Keski-Vakkuri, Phys. Rev. D 64 (2001) 044006, gr-qc/0005115.
[10] E.C. Vagenas, Phys. Lett. B 503 (2001) 399, hep-th/0012134;
E.C. Vagenas, Phys. Lett. B 559 (2003) 65, hep-th/0209185;
E.C. Vagenas, Nuovo Cimento B 117 (2002) 899, hep-th/0111047.
[11] A.J.M. Medved, E.C. Vagenas, Mod. Phys. Lett. A 20 (2005) 2449.
[12] M. Arzano, A.J.M. Medved, E.C. Vagenas, JHEP 0509 (2005) 037, hep-th/0505266.
[13] Q.Q. Jiang, S.Q. Wu, Phys. Lett. B 635 (2006) 151, hep-th/0511123;
Q.Q. Jiang, S.Q. Wu, X. Cai, Phys. Rev. D 73 (2006) 064003, hep-th/0512351.
[14] J.Y. Zhang, Z. Zhao, JHEP 0510 (2005) 055;
J.Y. Zhang, Z. Zhao, Nucl. Phys. B (2005) 725, 173.
[15] W.B. Liu, Phys. Lett. B 634 (2006) 541, gr-qc/0512099.
[16] R. Kerner, R.B. Mann, Phys. Rev. D 73 (2006) 104010;
R. Kerner, R.B. Mann, Phys. Rev. D 75 (2007) 084002.
[17] P. Mitra, Phys. Lett. B 648 (2007) 240;
B. Chatterjee, A. Ghosh, P. Mitra, arXiv: 0801.3583 [hep-th].
[18] L. Zhao, hep-th/0602065.
[19] Y. Sekiwa, arXiv: 0802.3266 [hep-th].
[20] R. Kerner, R.B. Mann, Class. Quantum Grav. 25 (2008) 095014, arXiv: 0710.0612 [hep-th].
[21] R. Li, J.R. Ren, Phys. Lett. B 661 (2008) 370, arXiv: 0802.3954 [hep-th].
[22] R.Di. Criscienzo, L. Vanzo, arXiv: 0803.0435 [hep-th].
[23] R. Li, J.R. Ren, S.W. Wei, arXiv: 0803.1410 [hep-th].
[24] R. Kerner, R.B. Mann, arXiv: 0803.2246 [hep-th].
[25] D.Y. Chen, Q.Q. Jiang, S.Z. Yang, X.T. Zu, arXiv: 0803.3248 [hep-th].
[26] D.Y. Chen, Q.Q. Jiang, X.T. Zu, arXiv: 0804.0131 [hep-th].
[27] D. Garfinkle, G.T. Horowitz, A. Strominger, Phys. Rev. D 43 (1991) 3140.
[28] G.T. Horowitz, J.M. Maldacena, A. Strominger, Phys. Lett. B 383 (1996) 151, hep-th/9603109.
[29] S. Gangopadhyay, S. Kulkarni, Phys. Rev. D 77 (2008) 024038, arXiv: 0710.0974 [hep-th].
[30] S.A. Hayward, R. Di Criscienzo, L. Vanzo, M. Nadalini, S. Zerbini, arXiv: 0806.0014 [gr-qc].
[31] H. Shin, W. Kim, JHEP 0706 (2007) 012, arXiv: 0705.0265 [hep-th].
[32] J. Maharana, J.H. Schwarz, Nucl. Phys. B 390 (1993) 3, hep-th/9207016.
[33] A. Sen, Nucl. Phys. B 404 (1993) 109, hep-th/9207053.
[34] S.Q. Wu, X. Cai, Gen. Relativ. Gravit. 33 (2001) 1181;
S.Q. Wu, X. Cai, Gen. Relativ. Gravit. 34 (2002) 605.